Analysis of Car Crash Simulation Data with Nonlinear Machine Learning Methods – Supplementary Material

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1 Reconstruction Error per Beam

Instead of using $k = 8$ clusters as we did in our main paper, we here provide the reconstruction error for PCA and PML for $k = 4$ clusters (k-means and spectral clustering) and for the 4 beams themselves. As in Sect. 2.5.1 of the main paper, we compare the reconstruction error per beam (2.8). The results are shown in Tables 1,2 and 3.

We directly see, that the reconstruction works best for the spectral clustering. Except for the one-dimensional PCA results, the clustering of the k-means algorithm leads to better or equal results than the standard subdivision into the four beams. Therefore, it is reasonable to consider the application of a clustering algorithm before running the dimensionality reduction algorithms.

Beam	1	2	3	4	Mean
PCA, $s = 1$	1.75	2.58	1.85	2.89	2.27
PML, $s = 1$	1.57	2.26	1.59	2.14	1.89
LLA, $s = 1$	0.68	0.85	0.83	1.18	0.89
PCA, $s = 2$	1.30	1.86	1.50	2.51	1.79
PML, $s = 2$	0.80	0.98	0.92	1.08	0.94
LLA, $s = 2$	0.55	0.80	0.77	0.86	0.75
PCA, $s = 3$	0.95	1.52	1.08	1.70	1.31
PML, $s = 3$	0.34	0.96	0.52	0.68	0.62
LLA, $s = 3$	0.56	0.65	0.68	0.73	0.65

Table 1: Error of principal component analysis (PCA) and principal manifold learning (PML) times 10^2 for each beam (beams as clusters).

Beam	1	2	3	4	Mean
PCA, $s = 1$	4.62	3.09	2.11	2.70	3.13
PML, $s = 1$	1.53	2.19	1.21	1.94	1.72
LLA, $s = 1$	0.71	1.35	0.53	1.00	0.90
PCA, $s = 2$	1.31	1.91	1.00	1.78	1.50
PML, $s = 2$	0.65	1.00	0.54	1.02	0.80
LLA, $s = 2$	0.59	0.99	0.48	0.92	0.74
PCA, $s = 3$	1.08	1.65	0.62	1.12	1.12
PML, $s = 3$	0.46	0.76	0.49	0.95	0.67
LLA, $s = 3$	0.55	0.97	0.43	0.80	0.69

Table 2: Error of principal component analysis (PCA) and principal manifold learning (PML) times 10^2 for each beam (k-means with 4 clusters).

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Beam	1	2	3	4	Mean
PCA, $s = 1$	1.78	2.51	1.66	2.50	2.11
PML, $s = 1$	1.48	2.12	1.28	1.90	1.70
LLA, $s = 1$	0.69	1.04	0.57	0.72	0.75
PCA, $s = 2$	1.23	1.84	0.95	1.55	1.39
PML, $s = 2$	0.61	0.98	0.48	0.73	0.70
LLA, $s = 2$	0.49	0.72	0.55	0.68	0.61
PCA, $s = 3$	0.99	1.49	0.52	0.71	0.93
PML, $s = 3$	0.39	0.62	0.37	0.50	0.47
LLA, $s = 3$	0.38	0.57	0.42	0.53	0.48

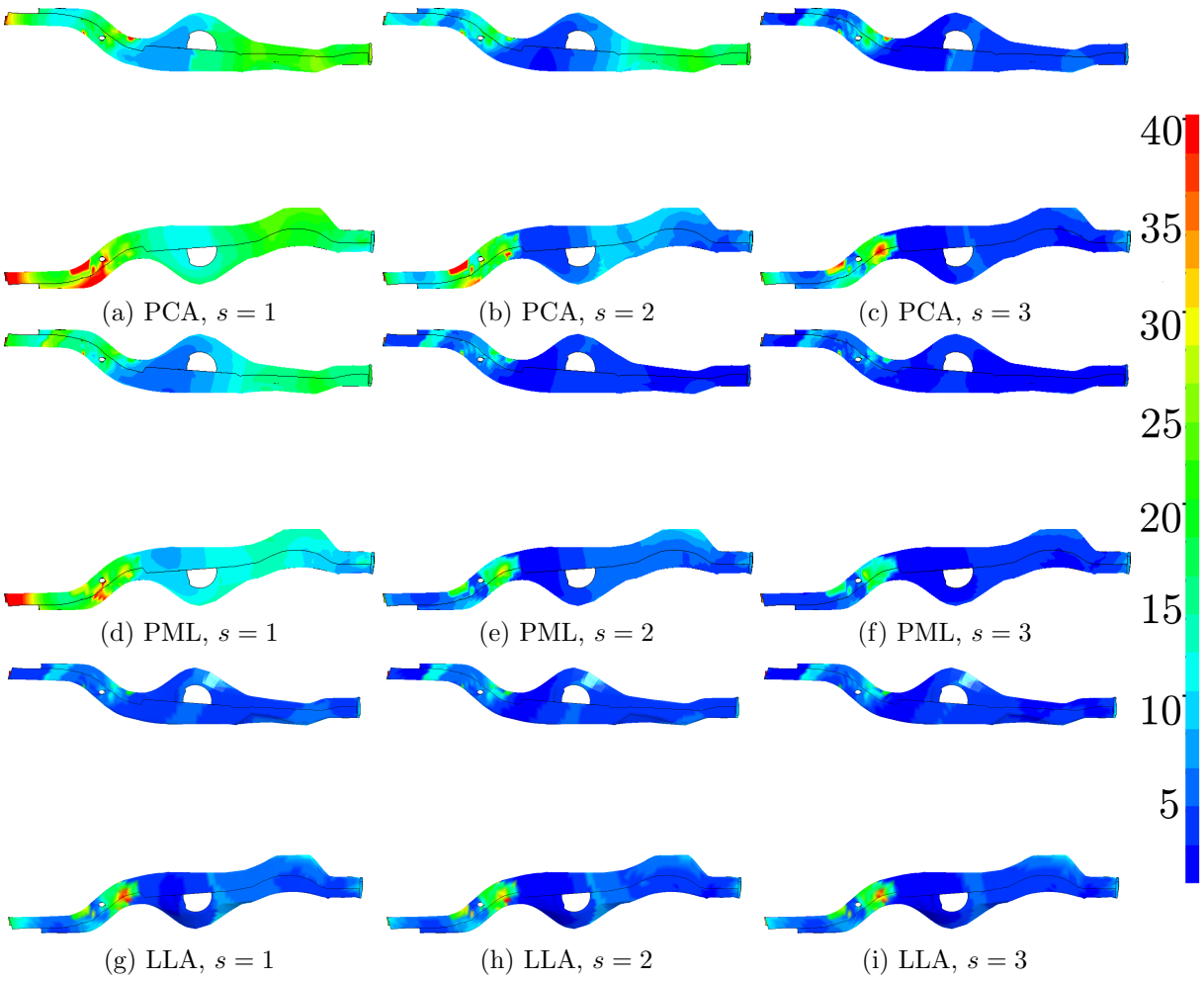
Table 3: Error of principal component analysis (PCA) and principal manifold learning (PML) times 10^2 for each beam (spectral clustering with 4 clusters).

2 Reconstruction Error per Node

In the main paper we omitted the plot for the reconstruction error per node for the spectral clustering algorithm because the results behaved similar to the results for the k-means clustering. For the sake of completeness, we present the omitted reconstruction error in Figure 1.

3 Latent Space Variable Separation

In Sect. 2.5.3 of the main paper, we showed the latent space variable separation for certain cluster choices for the diffusion maps algorithm. Although a clear separation with respect to the thickness of one of the car beams was visible, it was not shown that this nicely separated also the bending behavior of the beams. To this end, Figure 3 shows the simulation results corresponding to six data points for a three-dimensional embedding based on cluster 4 from the spectral clustering, where diffusion maps is applied for $t_j = 6$, i.e. the setting from Sect. 2.5.3. One observes, that the two groups of data in the lower dimensional embedding correspond to two distinctly different behaviors of the beams on one side of the truck.



(a) PCA, $s = 1$ (b) PCA, $s = 2$ (c) PCA, $s = 3$

(d) PML, $s = 1$ (e) PML, $s = 2$ (f) PML, $s = 3$

(g) LLA, $s = 1$ (h) LLA, $s = 2$ (i) LLA, $s = 3$

Figure 1: Mean of the squared l_2 errors of the reconstructed displacements over all test runs for the spectral clustering at the timesteps $t_i = 7$, $t_j = 6$ for PCA (top), PML (middle) and LLA (bottom). The latent space dimension s increases from left to right. Values greater than 40 are coloured red.

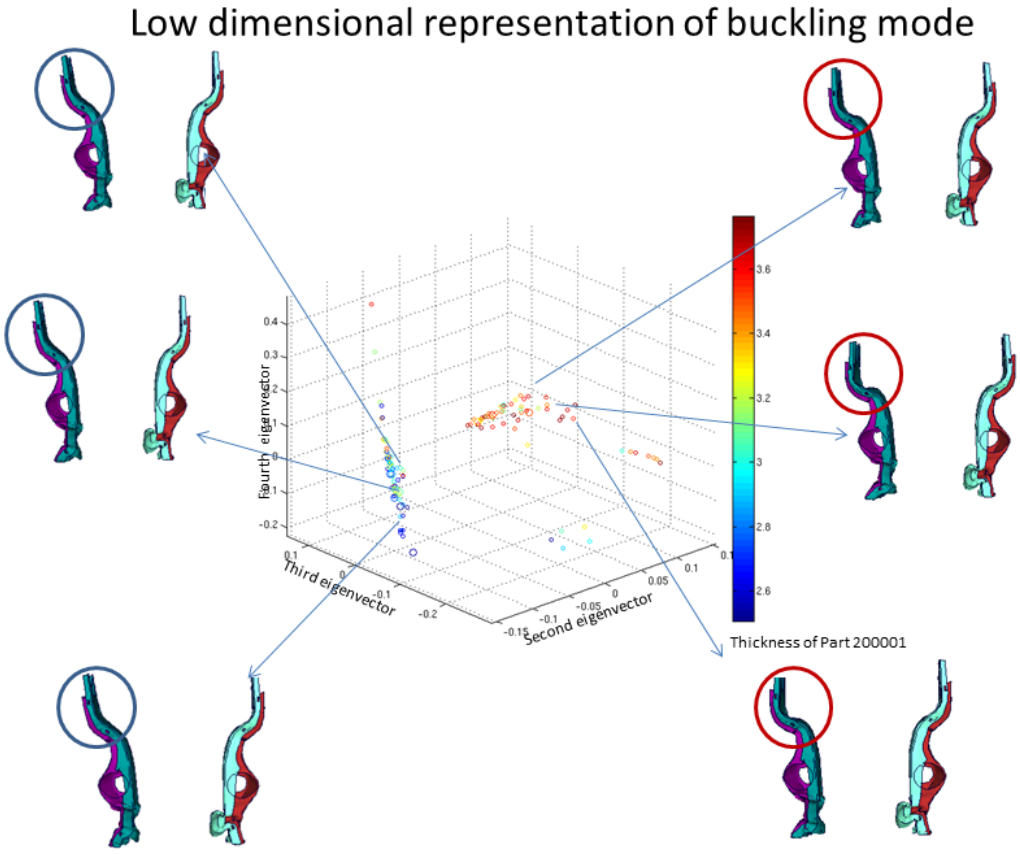


Figure 2: Separation of the bending behavior of the beams.